

Problem Set III: Due February 22, 2016

- 1.) Consider an ensemble of M stationary test particles in a plasma of N particles, $N \gg M$ and $n\lambda_D^3 > 1$. Calculate the electrostatic energy of the field produced by the M test particles, assuming their positions are uncorrelated.

- 2.) Derive the growth rate of the gentle bump-on-tail instability.
 - a.) Develop the system as an off-shoot of the beam-plasma system. Explain when kinetic processes become relevant.
 - b.) Derive the wave frequency and growth rate.

- 3.) Derive the growth rate of the current driven ion acoustic instability.
 - a.) Derive the ion acoustic wave kinetically. Calculate frequency and growth. Assume no electron current.
 - b.) Now allow electron current, so $\langle f_e \rangle$ is a shifted Maxwellian. When is instability possible? Take ions as unshifted Maxwellian and $\delta f_e = |e| \frac{\hat{\phi}}{T_e} \langle f_e \rangle + \hat{h}_e$.
 - c.) Derive the marginality condition for the CDIA. What parameters control stability?
 - d.) Calculate the electron and ion heating.

- 4.) a.) Consider a chunk of collisionless, self-gravitating matter in one dimension. Here, take a "chunk" to be:

$$f = \begin{cases} f_0, & u_0 - \Delta v < v < u_0 + \Delta v \\ 0, & \text{elsewhere} \end{cases}.$$

Here, f_0 is constant. Take $u_0, \Delta v$ fixed. Using the Vlasov-Poisson system, calculate the marginal stability criterion for Jeans instability. Compare your result to the case discussed in class for a self-gravitating gas.

- b.) Now consider a plasma, with

$$f = \begin{cases} f_{\max} + f_0, & u_0 - \Delta v < v < u_0 + \Delta v \\ f_{\max}, & \text{elsewhere} \end{cases}.$$

Consider $f_0 > 0$ and $f_0 < 0$. f_{\max} is the usual Maxwellian. Of course $f_{\max} + f_0 > 0$, for all v . What is the marginality condition now? Relate your result to the bunching condition discussed in class for the beam-plasma interaction. Hint: Consider the sign of the dielectric function.

- c.) For collisionless, self-gravitating matter with an initially Jeans unstable distribution, discuss how the instability might saturate. Hint: Consider simple quasi-linear analysis.
- 5.) Consider an electron and ion plasma which is stable, but in which the electrons carry a current, i.e. assume a drift u_0 . Take T_i finite.
- a.) What are the collective resonances? When are they weakly damped, and approaching marginality?
- b.) Estimate the thermal fluctuation spectrum ($\langle E^2 \rangle_{k,\omega} / 8\pi$) for the system described in a.).
- c.) *Quantitatively* discuss the breakdown of the test particle model assumptions as the system approaches marginality as the drift u_0 increases.

- 6.) For the system of Problem 5:
 - a.) Derive the rate of electron-ion momentum transfer. What are the key dimensionless parameters determining this? Assume parameters such that the system is stable.
 - b.) How does increasing drift affect the transfer? Assume the system remains stable, but approaches marginality from below.
- 7.) Read and summarize the posted article by Rostoker and Rosenbluth on the Test Particle Model. Describe the key ideas of the Test Particle Model and how they are developed.
- 8.) Read and summarize the posted article by Roberts and Nielson on Saturation of the Two-Stream Instability. Explain physically how saturation occurs.
- 9.) Calculate the electromagnetic energy spectrum of an electron plasma at thermal equilibrium at temperature T .
- 10.)
 - i.) How far does a plasma wave packet travel at thermal equilibrium?
 - ii.) Describe the electric field “wake” left by a test electron in a plasma.
 - iii.) Calculate the slowing down time for the particle by plasma wave emission.
 - iv.) How does the system off-set the natural slowing down of the particle by emission? Try to calculate this and compare it to losses due to emission.